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We study the collective quark excitations in the system of two interpenetrating streams of the quark-gluon plasma. In contrast to the gluon modes, all quark ones appear to be stable in such a system. The quark modes in the two-stream system are compared to those in the isotropic plasma.

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The quark-gluon plasma exhibits a rich spectrum of the collective excitations. During the last two decades a lot of efforts has been paid to study the equilibrium plasma and the excitations are rather well understood in this case, see [1] for an extensive review. Much less is known about the collective modes in the nonequilibrium quark-gluon plasma such as the parton system generated at the early stage of ultrarelativistic heavy ion collisions at RHIC or LHC. The parton momentum distribution is not isotropic but strongly elongated along the beam [2,3]. Therefore, specific color fluctuations, instead of being damped, can exponentially grow and noticeably influence the temporal evolution of the whole system. In a series of our papers [4–6] it has been argued that there are indeed very fast unstable transverse gluon modes in such a parton system. The analysis [5,6] has been performed within the semiclassical transport theory [7,8]. However, it has been later shown that one gets the equivalent results within the hard loop approach [9] extended to the anisotropic systems [10]. In the same paper [10], the quark self energy for an arbitrary momentum distribution has been computed at one loop level and the dispersion relation of quarks in the anisotropic plasma has been also briefly discussed. The aim of this note is to consider the quark modes in the extremely anisotropic plasma i.e. the system of two interpenetrating streams. The gluon modes have been discussed in such a system in the several papers [4,11–14] and the instabilities have been found. We show here that in contrast to the gluon excitations all quark modes in the two-stream system are stable and rather similar to those in the isotropic plasma.

The quark dispersion relations are determined by the poles of the quark propagator or equivalently are found as solutions of the equation

$$\det \left[ \not{k} - \Sigma(k^\mu) \right] = 0, \quad (1)$$

where  $k^\mu = (\omega, \mathbf{k})$  is the mode's four momentum; the mass of the bare quark is assumed to vanish. The quark self energy  $\Sigma(k^\mu)$  has been found [10] within the Hard Loop approximation at one loop level in the following form:

$$\Sigma(k^\mu) = \frac{g^2}{16} C_F \int \frac{d^3 p}{(2\pi)^3} \frac{f(\mathbf{p})}{|\mathbf{p}|} \frac{\gamma_0 - \mathbf{v} \cdot \boldsymbol{\gamma}}{\omega - \mathbf{v} \cdot \mathbf{k} + i0^+}, \quad (2)$$

where  $g$  is the coupling constant of QCD with  $N_c$  colors and  $C_F \equiv (N_c^2 - 1)/N_c$ ;  $\mathbf{v} \equiv \mathbf{p}/|\mathbf{p}|$  is the parton velocity and  $\gamma^\mu = (\gamma^0, \boldsymbol{\gamma})$  are Dirac matrices;  $f(\mathbf{p}) \equiv n(\mathbf{p}) + \bar{n}(\mathbf{p}) + 2n_g(\mathbf{p})$  and  $n(\mathbf{p})$ ,  $\bar{n}(\mathbf{p})$ , and  $n_g(\mathbf{p})$  denote distribution functions of, respectively, quarks, antiquarks and gluons. The effective distribution function  $f$  is arbitrary though symmetric i.e.  $f(-\mathbf{p}) = f(\mathbf{p})$ .

Since the spinor structure of  $\Sigma$  given by Eq. (2) is very simple i.e.  $\Sigma(k^\mu) = \gamma^0 \Sigma^0(\omega, \mathbf{k}) - \boldsymbol{\gamma} \boldsymbol{\Sigma}(\omega, \mathbf{k})$ , the dispersion equation (1) simplifies to

$$(\omega - \Sigma^0(\omega, \mathbf{k}))^2 - (\mathbf{k} - \boldsymbol{\Sigma}(\omega, \mathbf{k}))^2 = 0. \quad (3)$$

We model the system of two interpenetrating streams with the distribution function of the form

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$$f(\mathbf{p}) = (2\pi)^3 \rho [\delta^{(3)}(\mathbf{p} - \mathbf{q}) + \delta^{(3)}(\mathbf{p} + \mathbf{q})] , \quad (4)$$

where the parameter  $\rho$  is related to the parton density in the stream. The vector  $\mathbf{q}$  is assumed to be parallel to  $z$ -axis. Substituting (4) into Eq. (2) one immediately finds the self energies

$$\begin{aligned} \Sigma^0(\omega, \mathbf{k}) &= m^2 \frac{\omega}{\omega^2 - k_z^2} , \\ \Sigma^x(\omega, \mathbf{k}) &= \Sigma^y(\omega, \mathbf{k}) = 0 , \quad \Sigma^z(\omega, \mathbf{k}) = m^2 \frac{k_z}{\omega^2 - k_z^2} , \end{aligned}$$

where  $m^2 \equiv g^2 C_F \rho / |\mathbf{q}|$ , and gets the following dispersion equation:

$$\frac{\omega^2(\omega^2 - k_z^2 - m^2)^2}{(\omega^2 - k_z^2)^2} - k_T^2 - \frac{k_z^2(\omega^2 - k_z^2 - m^2)^2}{(\omega^2 - k_z^2)^2} = 0 , \quad (5)$$

with  $k_T \equiv \sqrt{k_x^2 + k_y^2}$ . Eq. (5) is easily solved providing two quark ( $\omega_{\pm} > 0$ ) and two antiquark ( $\omega_{\pm} < 0$ ) modes:

$$\omega_{\pm}^2(k_T, k_z) = m^2 + k_z^2 + \frac{1}{2}k_T^2 \pm \sqrt{m^2 k_T^2 + \frac{1}{4}k_T^4} \cong m^2 + k_z^2 + \frac{1}{2}k_T^2 \pm m k_T , \quad (6)$$

where the second approximate equality holds for  $m \gg k_T$ . Since the modes are pure real, there is no instability. The dispersion relations (6) are illustrated in Figs. 1 and 2.

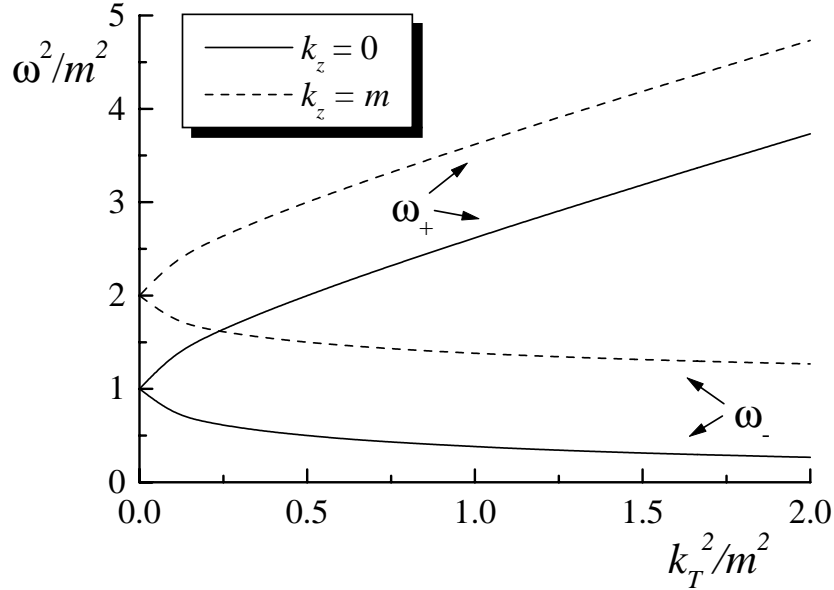


FIG. 1.  $\omega_{\pm}^2$  as a function of  $k_T$  for  $k_z = 0$  and  $k_z = m$ .

Let us now compare the quark modes in the two-stream system (6) to those in the isotropic plasma studied, in particular, in [15,16]. In contrast to the derivations in [15,16] we do not assume that the system is in the thermodynamic equilibrium. We only demand the isotropy of the parton momentum distribution i.e. the distribution function which enters the self energy (2) is of the form  $f(\mathbf{p}) = f(|\mathbf{p}|)$ . Then, the angular and momentum integrals in Eq. (2) factorize and one finds

$$\Sigma^0(\omega, \mathbf{k}) = \frac{M^2}{2k} \left[ \ln \left| \frac{\omega + k}{\omega - k} \right| - i\pi \Theta(k - \omega) \right] = \frac{M^2}{\omega} (1 + \mathcal{O}(k^2/\omega^2)) , \quad (7)$$

$$\Sigma(\omega, \mathbf{k}) = \frac{M^2}{k^2} \mathbf{k} - \frac{M^2 \omega}{2k^3} \left[ \ln \left| \frac{\omega + k}{\omega - k} \right| - i\pi \Theta(k - \omega) \right] \mathbf{k} = \mathcal{O}(k^2/\omega^2) \frac{M^2}{k^2} \mathbf{k} , \quad (8)$$

where  $k \equiv |\mathbf{k}|$  and

In the case of massless

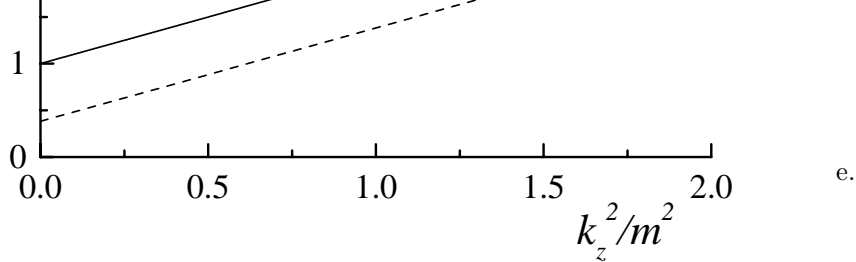


FIG. 2.  $\omega_{\pm}^2$  as a function of  $k_z$  for  $k_T = 0$  and  $k_T = m$ .

Substituting the self energies (7,8) into the dispersion equation (3), one finds the quark modes which in the long wavelength limit ( $\omega \gg k$ ) can be computed analytically and read

$$\omega_{\pm}^2(k) \cong M^2 + \frac{1}{2}k^2 \pm M k. \quad (9)$$

As seen, the quark long wavelength modes in the two-stream system (6) and in the isotropic plasma (9) are very similar to each other. The  $\omega_+$  mode is nearly the same in the two systems for any wavelength. However, there is a qualitative difference in the case of the  $\omega_-$  mode. In equilibrium,  $\omega_-$  initially decreases with  $k$  according to Eq. (9), then there is a shallow minimum at  $k \approx M$  and further  $\omega_-$  monotonically grows. In the two-stream system,  $\omega_-$  is monotonically decreasing function of  $k_T$ . Since the phase velocity of the  $\omega_-$  mode is greater than the velocity of light for sufficiently large  $k_T$ , the quasiquarks can emit gluons due to the Cherenkov mechanism. The relevance of the phenomenon for the nonequilibrium plasma from the early stage of heavy-ion collisions at RHIC and LHC is under study.

We conclude our considerations as follows. In contrast to the gluon modes, the quark ones are all stable in the two-stream system. In the long wavelength limit, the quasi quark properties in the two-stream and isotropic systems are qualitatively the same. The  $\omega_-$  mode can be responsible for the Cherenkov radiation.

- [1] J.-P. Blaizot and E. Iancu, hep-ph/0101103, to appear in Phys. Rept.
- [2] K. Geiger, Phys. Rep. **258**, 237 (1995).
- [3] X.-N. Wang, Phys. Rep. **280**, 287 (1997).
- [4] St. Mrówczyński, Phys. Lett. **B214** (1988) 587.
- [5] St. Mrówczyński, Phys. Lett. B **314**, 118 (1993); *ibid.* B **393**, 26 (1997).
- [6] St. Mrówczyński, Phys. Rev. C **49**, 2191 (1994).
- [7] H.-Th. Elze and U. Heinz, Phys. Rep. **183**, 81 (1989).
- [8] St. Mrówczyński, Phys. Rev. D **39**, 1940 (1989).
- [9] E. Braaten and R.D. Pisarski, Nucl. Phys. B **337**, 569 (1990).
- [10] St. Mrówczyński and M.H. Thoma, Phys. Rev. D **62**, 036011 (2000).
- [11] Yu. E. Pokrovskii and A.V. Selikhov, Pis'ma Zh. Eksp. Teor. Fiz. **47** 11 (1988).
- [12] Yu. E. Pokrovskii and A.V. Selikhov, Yad. Fiz. **52** 229 (1990).
- [13] Yu. E. Pokrovskii and A.V. Selikhov, Yad. Fiz. **52** 605 (1990).
- [14] O.P. Pavlenko, Yad. Fiz. **55** 2239 (1992).
- [15] H.A. Weldon, Phys. Rev. D **26**, 2789 (1982).
- [16] J.-P. Blaizot and J.-Y. Ollitrault, Phys. Rev. D **48**, 1390 (1993).